Rules of Inference

Rules of inference provide the justification of the steps used in a proof.

One important rule is called **modus ponens** or the **law of detachment**. It is based on the tautology $(p \land (p \rightarrow q)) \rightarrow q$. We write it in the following way:

The two hypotheses p and $p \rightarrow q$ are written in a column, and the conclusion 9 below a bar, where \therefore means "therefore".

∴ q

 $p \rightarrow$

p

Rules of Inference

The general form of a rule of inference is:

Р1 Р2

•

pn

∴ q

The rule states that if p_1 and p_2 and ... and p_n are all true, then q is true as well.

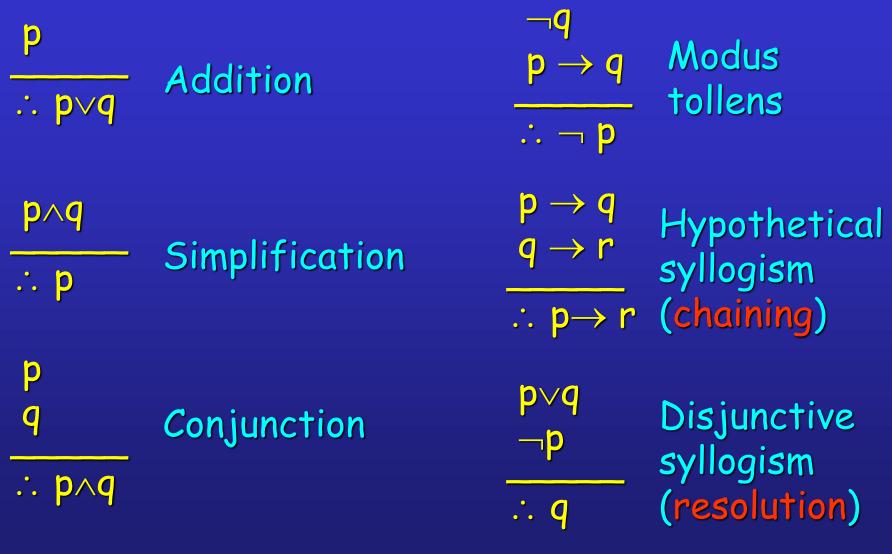
Each rule is an established tautology of $p_1 \land p_2 \land ... \land p_n \rightarrow q$

These rules of inference can be used in any mathematical argument and do not require any proof.

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Rules of Inference



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Just like a rule of inference, an **argument** consists of one or more hypotheses (or premises) and a conclusion.

We say that an argument is valid, if whenever all its hypotheses are true, its conclusion is also true. However, if any hypothesis is false, even a valid argument can lead to an incorrect conclusion.

Proof: show that hypotheses \rightarrow conclusion is true using rules of inference

Example:

"If 101 is divisible by 3, then 101² is divisible by 9. 101 is divisible by 3. Consequently, 101² is divisible by 9."

Although the argument is valid, its conclusion is incorrect, because one of the hypotheses is false ("101 is divisible by 3.").

If in the above argument we replace 101 with 102, we could correctly conclude that 102² is divisible by 9.

Which rule of inference was used in the last argument?

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p: "101 is divisible by 3."
q: "101<sup>2</sup> is divisible by 9."
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\begin{array}{c} p \\ p \rightarrow q \\ \hline \ddots q \end{array} \qquad Modus
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Unfortunately, one of the hypotheses (p) is false. Therefore, the conclusion q is incorrect.

Another example:

"If it rains today, then we will not have a barbeque today. If we do not have a barbeque today, then we will have a barbeque tomorrow. Therefore, if it rains today, then we will have a barbeque tomorrow."

This is a valid argument: If its hypotheses are true, then its conclusion is also true.

Let us formalize the previous argument: p: "It is raining today." q: "We will not have a barbecue today." r: "We will have a barbecue tomorrow." So the argument is of the following form:

 $\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \therefore P \rightarrow r \end{array} \qquad \begin{array}{l} \text{Hypothetical} \\ \text{syllogism} \end{array}$

Another example:

Gary is either intelligent or a good actor. If Gary is intelligent, then he can count from 1 to 10. Gary can only count from 1 to 3. Therefore, Gary is a good actor.

i: "Gary is intelligent."
a: "Gary is a good actor."
c: "Gary can count from 1 to 10."

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Step 1: ¬ c Step 2: i→ c Step 3: ¬ i Step 4: a∨i Step 5: a Hypothesis Hypothesis Modus tollens Steps 1 & 2 Hypothesis Disjunctive Syllogism Steps 3 & 4

Conclusion: a ("Gary is a good actor.")

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Yet another example:

If you listen to me, you will pass CS 320. You passed CS 320. Therefore, you have listened to me.

Is this argument valid?

No, it assumes $((p \rightarrow q) \land q) \rightarrow p$. This statement is not a tautology. It is **false** if p is false and q is true.